Chern-Simons as a geometrical setup for three-dimensional gauge theories

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I. INTRODUCTION

In a previous paper [1], it has been observed that the topological three-dimensional massive Yang-Mills gauge theory whose expression is given by the sum of the Yang-Mills action and of the Chern-Simons term [2]

$$\mathcal{S}_{YM}(A)+\mathcal{S}_{CS}(A), \tag{1.1}$$

with

$$\mathcal{S}_{YM}(A)=\frac{1}{4m}\mathrm{tr}\int d^3x F_{\mu\nu}F^{\mu\nu}, \tag{1.2}$$

and

$$\mathcal{S}_{CS}(A)=\tfrac{1}{2}\ \mathrm{tr}\int d^3x\varepsilon^{\mu\nu\rho}(A_\mu\partial_\nu A_\rho+\tfrac{2}{3}gA_\mu A_\nu A_\rho), \tag{1.3}$$

can be cast in the form of a pure Chern-Simons action through a nonlinear redefinition of the gauge connection, namely

$$\mathcal{S}_{YM}(A)+\mathcal{S}_{CS}(A)=\mathcal{S}_{CS}(\hat{A}), \tag{1.4}$$

with

$$\hat{A}_\mu=A_\mu+\sum_{n=1}^{\infty}\frac{1}{m^n}\vartheta_\mu^n(D,F). \tag{1.5}$$

The coefficients $\vartheta_\mu^n(D,F)$ in Eq. (1.5) turn out to be *local* and *covariant*, meaning that they are built only with the field strength[1] $F_{\mu\nu}$,

$$F_{\mu\nu}=\partial_\mu A_\nu-\partial_\nu A_\mu+g[A_\mu,A_\nu], \tag{1.6}$$

and the covariant derivative D_μ

$$D_\mu=\partial_\mu+g[A_\mu,]. \tag{1.7}$$

The two parameters g,m in the above expressions identify the gauge coupling constant and the so called topological mass [2]. According to the parametrization chosen for the topological massive Yang-Mills action (1.1), we can assign mass dimension 1 to the gauge field A_μ, so that the parameters g,m are, respectively, of mass dimension 0 and 1.

For instance, for the first four coefficients of the expansion (1.5), we have [1]

$$\begin{aligned}
\vartheta_\mu^1&=\tfrac{1}{4}\varepsilon_{\mu\sigma\tau}F^{\sigma\tau},\\
\vartheta_\mu^2&=\tfrac{1}{8}D^\sigma F_{\sigma\mu},\\
\vartheta_\mu^3&=-\frac{1}{16}\varepsilon_{\mu\sigma\tau}D^\sigma D_\rho F^{\rho\tau}+\frac{g}{48}\varepsilon_{\mu\sigma\tau}[F^{\sigma\rho},F_\rho^{\tau}],\\
\vartheta_\mu^4&=-\frac{5}{128}D^2D^\rho F_{\rho\mu}+\frac{5}{128}D^\nu D_\mu D^\lambda F_{\lambda\nu}\\
&\quad-\frac{7}{192}g[D^\rho F_{\rho\tau},F_\mu^{\tau}]-\frac{1}{48}g[D_\nu F_{\mu\lambda},F^{\lambda\nu}].
\end{aligned} \tag{1.8}$$

This work attempts to provide a detailed and self-contained cohomological analysis of the Eq. (1.4) and of the covariant character of the coefficients $\vartheta_\mu^n(D,F)$. Furthermore, the formulas (1.4), (1.5) will be generalized to any local[2] gauge invariant Yang-Mills type action:[3] $\int FD^2F$, etc. These features will enable us to interpret the topological

[1] As usual the gauge field A_μ is meant to be Lie algebra valued, $A_\mu=A_\mu^aT^a$, T^a being the anti-Hermitian generators of a semisimple Lie group.

[2] As we shall see in the Sec. V, a rather large class of nonlocal gauge invariant actions can be contemplated as well.

[3] According to the BRST analysis of gauge theories [3–5], the name Yang-Mills type action is employed here to denote a generic integrated local invariant polynomial with vanishing ghost number built only with the field strength F and its covariant derivatives. The corresponding actions are of a very different nature with respect to the Chern-Simons term, which is known to be BRST invariant only up to a total derivative. It belongs thus to the so called BRST cohomology modulo d, d being the space-time differential. Although the present considerations are referred to the flat Euclidean space-time, it is worth recalling that while the Chern-Simons term turns out to be metric independent, when defined on curved spaces, the Yang-Mills type actions couple in a nontrivial and direct way to the space-time metric [4].

Chern-Simons term as a gauge invariant functional acting on a suitably defined space of gauge connections. Any given local Yang-Mills type action is thus obtained by evaluating the Chern-Simons functional at a specific point of this space, yielding thus a pure geometrical set up for the three-dimensional gauge theories.

The work is organized as follows. In Sec. II we review the Becchi-Rouet-Stora-Tyutin (BRST) cohomology of three-dimensional gauge theories in the presence of the Chern-Simons action. In Sec. III we establish some general geometrical features of the topological Chern-Simons term which will account for the covariant character of the coefficients ϑ_μ^n and for the Eq. (1.4). Section IV deals with the generalization to a generic Yang-Mills type action. Although the content of this paper refers mainly to geometrical aspects, Sec. V will be devoted to the consequences following from the relation (1.4) at the quantum level. Further possible applications will be also outlined.

II. BRST COHOMOLOGY OF YANG-MILLS THEORY IN THE PRESENCE OF THE CHERN-SIMONS TERM

A. Generalities

The BRST cohomology of the Yang-Mills gauge theories has been studied extensively in the last years. Very general results and theorems have been established in any space-time dimension [3–5], being easily adapted to the present case. Following the standard procedure [6,7], the BRST differential s corresponding to the topological massive Yang-Mills action (1.1) is given by

$$
\begin{aligned}
sA_\mu &= D_\mu c, \\
sc &= -gc^2, \\
sA_\mu^* &= \frac{1}{2}\varepsilon_{\mu\nu\rho}F^{\nu\rho} + \frac{1}{m}D^\nu F_{\mu\nu} - g\{A_\mu^*, c\}, \\
sc^* &= D^\mu A_\mu^* + g[c^*, c],
\end{aligned} \tag{2.1}
$$

with c being the Faddeev-Popov ghost and (A_μ^*, c^*) identifying the two antifields needed in order to implement in cohomology [5] the equations of motion stemming from the action (1.1). The fields and antifields (A_μ, c, A_μ^*, c^*) carry, respectively, ghost number $(0,1,-1,-2)$.

In order to provide a cohomological understanding of the Eq. (1.4), we have first to specify the appropriate functional space for the BRST differential. As suggested by the Eq. (1.5), the latter will be identified with the space of the integrated local polynomials in the fields and antifields of arbitrary dimension. More precisely, the operator s will be allowed to act on the functional space of the integrated local formal power series in the fields and antifields. This choice is the most suitable one in view of the generalization of the Eq. (1.4) to higher order Yang-Mills type actions, $\int FD^2F$, $\int FD^2D^2F$, etc., which will be discussed later on. These terms fit naturally in the space of the local formal power series in the fields and antifields. Observe also that the inverse of the topological mass can be interpreted as the expansion parameter for the formal power series belonging to this functional space, as in the case of the coefficients ϑ_μ^n of the nonlinear field redefinition (1.5).

It is rather simple now to convince ourselves that, within the space of the local formal power series, the presence of the Chern-Simons term in the initial action (1.1) allows us to implement a recursive procedure which trivializes any BRST invariant term containing only F and its covariant derivatives. In order to have a direct and simple idea of the meaning of this statement, it is sufficient to consider the so called Abelian approximation of the BRST transformations (2.1), namely

$$
s \to s_0, \tag{2.2}
$$

with

$$
\begin{aligned}
s_0 A_\mu &= \partial_\mu c, \\
s_0 c &= 0, \\
s_0 A_\mu^* &= \frac{1}{2}\varepsilon_{\mu\nu\rho}F^{0\nu\rho} + \frac{1}{m}\partial^\nu F_{\mu\nu}^0, \\
s_0 c^* &= \partial^\mu A_\mu^*,
\end{aligned} \tag{2.3}
$$

and

$$
F_{\mu\nu}^0 = \partial_\mu A_\nu - \partial_\nu A_\mu . \tag{2.4}
$$

From Eqs. (2.3) we see that the s_0-transformations of the fields and antifields correspond to the case in which all (anti)commutators have been discarded, reducing thus to a set of Abelian transformations. The operator s_0 is actually the first term of the decomposition of the full BRST differential s according to the filtering operator [8,7]

$$
\mathcal{N} = \mathrm{tr}\int d^3x \left(A_\mu \frac{\delta}{\delta A_\mu} + c\frac{\delta}{\delta c} + A_\mu^* \frac{\delta}{\delta A_\mu^*} + c^* \frac{\delta}{\delta c^*} \right). \tag{2.5}
$$

As it is well known, the relevance of the operator s_0 is due to a very general theorem [8,7] on the BRST cohomology which states that the cohomology of the complete BRST differential s is isomorphic to a subspace of the cohomology of the operator s_0. This implies, in particular, that if the cohomology of s_0 is trivial, that of the full operator s will be empty as well.

Let us now proceed by rewriting the third equation of Eq. (2.3) in the following form

$$
F_{\mu\nu}^0 = s_0(\varepsilon_{\mu\nu\rho}A^{*\rho}) - \frac{1}{m}\varepsilon_{\mu\nu\rho}\partial_\lambda F^{0\rho\lambda}, \tag{2.6}
$$

where use has been made of the Euclidean normalization

$$
\varepsilon^{\mu\nu\rho}\varepsilon_{\mu\sigma\tau} = \delta_\sigma^\nu \delta_\tau^\rho - \delta_\tau^\nu \delta_\sigma^\rho . \tag{2.7}
$$

From the Eq. (2.6) we see that we can replace the field strength $F_{\mu\nu}^0$ by a pure BRST variation with in addition a term of higher dimension containing a space-time derivative and a factor $1/m$. The Eq. (2.6) has the meaning of a recursive formula since $F^0_{\mu\nu}$ appears on both sides, thereby allowing us to express $F^0_{\mu\nu}$ as a pure s_0-variation: i.e.,

$$
\begin{aligned}
F^0_{\mu\nu} &= s_0(\varepsilon_{\mu\nu\rho}A^{*\rho}) - \frac{1}{m}\varepsilon_{\mu\nu\rho}\partial_\lambda F^{0\rho\lambda} \\
&= s_0\left(\varepsilon_{\mu\nu\rho}A^{*\rho} - \frac{1}{m}(\partial_\mu A^*_\nu - \partial_\nu A^*_\mu)\right) \\
&\quad + \frac{1}{m^2}(\partial_\mu\partial^\sigma F^0_{\nu\sigma} - \partial_\nu\partial^\sigma F^0_{\mu\sigma}) \\
&= s_0\left(\varepsilon_{\mu\nu\rho}A^{*\rho} - \frac{1}{m}(\partial_\mu A^*_\nu - \partial_\nu A^*_\mu)\right. \\
&\quad \left. + \frac{1}{m^2}(\varepsilon_{\mu\nu\sigma}\partial_\rho - \varepsilon_{\mu\nu\rho}\partial_\sigma)\partial^\sigma A^{*\rho}\right) \\
&\quad - \frac{1}{m^3}(\varepsilon_{\mu\nu\sigma}\partial_\rho - \varepsilon_{\mu\nu\rho}\partial_\sigma)\partial^\sigma\partial_\lambda F^{0\rho\lambda} \\
&= s_0\left(\varepsilon_{\mu\nu\rho}A^{*\rho} - \frac{1}{m}(\partial_\mu A^*_\nu - \partial_\nu A^*_\mu)\right. \\
&\quad + \frac{1}{m^2}(\varepsilon_{\mu\nu\sigma}\partial_\rho - \varepsilon_{\mu\nu\rho}\partial_\sigma)\partial^\sigma A^{*\rho} - \frac{1}{m^3} \\
&\quad \left. \times(\varepsilon_{\mu\nu\sigma}\partial_\rho - \varepsilon_{\mu\nu\rho}\partial_\sigma)\varepsilon^{\rho\lambda\tau}\partial^\sigma\partial_\lambda A^*_\tau\right) + O\left(\frac{1}{m^4}\right) \\
&= \ldots\ldots\ldots\ldots\ldots .
\end{aligned}
\tag{2.8}
$$

It becomes now apparent that this iterative procedure will result in a formal power series in the expansion parameter $1/m$ whose coefficients will contain only the antifield A^*_μ and its space-time derivatives. The above formula expresses the triviality of the field strength $F^0_{\mu\nu}$. As a consequence, any invariant local term depending only on $F^0_{\mu\nu}$ and its space-time derivatives can be written as a pure s_0-variation. Of course, the same property holds at the level of the full BRST operator s with the result that all the invariant local terms made up with the field strength $F_{\mu\nu}$ and its covariant derivatives can be cast in the form of an exact BRST variation of a local formal power series. Therefore, from the general results on the BRST cohomology of gauge theories [3–5], we can infer that in the space of the integrated local power series in the fields and antifields, the unique nontrivial element with the quantum numbers of an action can be identified with the pure topological Chern-Simons term[4] (see also Secs. V and VI of Ref. [9]).

[4]We recall here that, from the general theorems proven in [5], the antifields do not contribute to the BRST cohomology in the sector of zero ghost number for Yang-Mills gauge theories with semi-simple group in arbitrary space-time dimension.

It is worth recalling that, within the BRST algebraic framework, the terms of the action which are exact turn out to correspond to pure field redefinitions. Therefore, the formulas (1.4), (1.5) arise as a consequence of the BRST triviality of the Yang-Mills term. We also underline that the possibility of rewriting the Yang-Mills action in exact form relies crucially on the presence of the topological Chern-Simons term in the starting action. As one can easily understand, this is due to the fact that the field variation of Chern-Simons yields the (dual) of the field strength $F_{\mu\nu}$, as expressed by the BRST transformation of the antifield A^*_μ in Eqs. (2.1). Without the presence of the term $\varepsilon_{\mu\nu\rho}F^{\nu\rho}$ in the right hand side of Eqs. (2.1), it would be impossible to implement the previous recursive procedure, as the left hand side of the Eq. (2.6) would be vanishing. The formula (2.6) would become thus useless. This means that if the Chern-Simons term is not included in the initial action, there is no way of (re)expressing the Yang-Mills action in the form of an exact variation of a local formal power series. However, as soon as the topological Chern-Simons is turned on, we can immediately reabsorb the Yang-Mills term through a nonlinear field redefinition.

B. Complete ladder structure

The previous cohomological considerations can be understood in a simple way by noticing that the transformations (2.3) can be cast in a form which is typical of the topological theories of the Schwartz type [10], as for instance pure Chern-Simons. In fact, using as new variables the redefined antifields

$$
\begin{aligned}
\tilde{A}^*_{\mu\nu} &= A^*_{\mu\nu} - \frac{1}{m}\varepsilon_{\mu\nu\rho}\partial_\sigma A^{*\rho\sigma} - \frac{1}{m^2}\partial^2 A^*_{\mu\nu} \\
&\quad + \frac{1}{m^3}\varepsilon_{\mu\nu\rho}\partial^2\partial_\sigma A^{*\rho\sigma} + \frac{1}{m^4}\partial^2\partial^2 A^*_{\mu\nu} + O(1/m^5),
\end{aligned}
\tag{2.9}
$$

$$\tilde{c}^* = c^* - \frac{1}{m^2}\partial^2 c^* + \frac{1}{m^4}\partial^2\partial^2 c^* + O(1/m^5),$$

with

$$A^*_{\mu\nu} = \varepsilon_{\mu\nu\rho}A^{*\rho}, \tag{2.10}$$

one easily gets, up to the order $1/m^5$,

$$
\begin{aligned}
s_0 A_\mu &= \partial_\mu c, \\
s_0 c &= 0, \\
s_0 \tilde{A}^*_{\mu\nu} &= F^0_{\mu\nu}, \\
s_0 \tilde{c}^* &= \tfrac{1}{2}\varepsilon^{\mu\nu\rho}\partial_\mu \tilde{A}^*_{\nu\rho}.
\end{aligned}
\tag{2.11}
$$

This structure, called complete ladder structure [7], implies that all fields, but the undifferentiated ghost c can be grouped in BRST doublets, meaning that the cohomology of s_0 in the space of the formal power series is spanned by polynomials in the undifferentiated ghost c. As it is well known this result, combined with the requirement of the rigid gauge invariance [3,5], allows us to identify the cohomology classes of the full BRST differential with the invariant polynomials in the undifferentiated Faddeev-Popov ghost c built with monomials of the kind tr c^{2n+1}, $n\geqslant 1$. It follows then that the cohomology of s modulo d in the sector of the local power series with the same quantum numbers of a Lagrangian has a unique nontrivial element, corresponding (via descent equations [7]) to the ghost monomial tr c^3. The resulting action is the Chern-Simons term.

Having justified the Eqs. (1.4), (1.5), let us now turn to the covariant character of the coefficients ϑ^n_μ in Eqs. (1.5). This will be the task of the next section.

III. SOME USEFUL PROPERTIES OF THE PURE CHERN-SIMONS ACTION

In order to account for the covariant character of the coefficients ϑ^n_μ in the Eq. (1.5), we recall first some simple properties of the Chern-Simons term. Let A_μ be a given gauge connection and let $\mathcal{S}_{CS}(A)$ be the corresponding BRST invariant Chern-Simons action, as given by the expression (1.3). Let us now vary the gauge field A_μ by an arbitrary amount δA_μ and let us try to establish the transformation law for δA_μ in order that the new Chern-Simons functional $\mathcal{S}_{CS}(\hat{A})$ evaluated at $\hat{A}_\mu = A_\mu + \delta A_\mu$, i.e.

$$\mathcal{S}_{CS}(\hat{A}) = \mathcal{S}_{CS}(A) + \mathrm{tr}\int d^3x\,\varepsilon^{\mu\nu\rho}\left(\frac{1}{2}\delta A_\mu F_{\nu\rho} + \frac{1}{2}\delta A_\mu D_\nu \delta A_\rho + \frac{g}{3}\delta A_\mu \delta A_\nu \delta A_\rho\right), \tag{3.1}$$

is still BRST invariant. Notice also that the variation δA_μ is not treated as an *infinitesimal* quantity, the formula (3.1) being indeed exact.

Requiring then that

$$s\mathcal{S}_{CS}(\hat{A}) = 0, \tag{3.2}$$

and recalling that

$$s\mathcal{S}_{CS}(A) = 0, \tag{3.3}$$

we easily obtain

$$\begin{aligned} 0 = \mathrm{tr}\int d^3x\,\varepsilon^{\mu\nu\rho}\{&(s\,\delta A_\mu - g[\delta A_\mu, c])F_{\nu\rho} \\ &+ 2g(s\,\delta A_\mu)\,\delta A_\nu \delta A_\rho + [(s\,\delta A_\mu) D_\nu \delta A_\rho \\ &+ \delta A_\mu s(D_\nu \delta A_\rho)]\}. \end{aligned} \tag{3.4}$$

The condition (3.4) implies that

$$s\,\delta A_\mu = g[\delta A_\mu, c], \tag{3.5}$$

meaning thus that δA_μ transforms covariantly. From the Eq. (3.5) it follows that the modified field $\hat{A}_\mu = A_\mu + \delta A_\mu$ is a *connection*,

$$s\hat{A}_\mu = \partial_\mu c + g[\hat{A}_\mu, c], \tag{3.6}$$

as it should be.

We see therefore that if we vary the gauge field A_μ by an arbitrary amount δA_μ which transforms covariantly under BRST, the resulting Chern-Simons term $\mathcal{S}_{CS}(A_\mu + \delta A_\mu)$ will remain gauge invariant. Of course, the covariant character persists also in the case in which δA_μ is meant to be a local formal power series in the expansion parameter $1/m$, i.e.

$$\delta A_\mu = \sum_{n=1}^{\infty} \frac{1}{m^n}\vartheta^n_\mu. \tag{3.7}$$

From the Eq. (3.5) we have

$$s\,\vartheta^n_\mu = g[\vartheta^n_\mu, c], \tag{3.8}$$

owing to the fact that coefficients ϑ^n_μ with different values of n have to be considered independent, being of different mass dimensions.

Moreover, from the Eq. (3.7) we obviously get

$$\mathcal{S}_{CS}(A_\mu + \delta A_\mu) = \mathcal{S}_{CS}(A) + \sum_{n=1}^{\infty}\frac{1}{m^n}\mathcal{S}^n, \tag{3.9}$$

$\mathcal{S}^n$ being integrated local formal power series corresponding to the expansion of $\mathcal{S}_{CS}(A_\mu + \delta A_\mu)$ in powers of the inverse of the topological mass m, according to Eq. (3.7).

Furthermore, from the BRST invariance of $\mathcal{S}_{CS}(A_\mu + \delta A_\mu)$ and of $\mathcal{S}_{CS}(A)$, we have

$$s\mathcal{S}^n = 0, \tag{3.10}$$

implying that the coefficients $\mathcal{S}^n$ in the Eq. (3.9) are BRST invariant. Recalling now that the Chern-Simons term is the unique nontrivial action in the space of the local formal power series, it follows that the $\mathcal{S}^n$'s in Eq. (3.9) have to be necessarily BRST exact, namely

$$\mathcal{S}^n = s\hat{\mathcal{S}}^n, \tag{3.11}$$

for some local integrated formal power series $\hat{\mathcal{S}}^n$ with negative ghost number. We are now ready to give a cohomological proof of the Eq. (1.4).

In fact, the following Lemma holds:

Lemma. Among the class of the BRST invariant Chern-Simons functionals $\mathcal{S}_{CS}(\hat{A})$

$$\mathcal{S}_{CS}(\hat{A})=\tfrac{1}{2}\ \mathrm{tr}\int d^3x\varepsilon^{\mu\nu\rho}(\hat{A}_\mu\partial_\nu\hat{A}_\rho+\tfrac{2}{3}g\hat{A}_\mu\hat{A}_\nu\hat{A}_\rho), \tag{3.12}$$

$\hat{A}_\mu$ being a connection of the type

$$\hat{A}_\mu=A_\mu+\sum_{n=1}^{\infty}\frac{1}{m^n}\vartheta^n_\mu, \tag{3.13}$$

it is always possible to find a set of covariant[5] coefficients ϑ^n_μ such that

$$\mathcal{S}_{CS}(\hat{A})=\mathcal{S}_{CS}(A)+\frac{1}{4m}\mathrm{tr}\int d^3xF_{\mu\nu}F^{\mu\nu}. \tag{3.14}$$

Proof. In order to prove the Lemma, we proceed by assuming the converse, as it is usual in this kind of problem. Let us suppose then that the Eq. (3.14) does not hold, i.e. that

$$\mathcal{S}_{CS}(\hat{A})\neq\mathcal{S}_{CS}(A)+\frac{1}{4m}\mathrm{tr}\int d^3xF_{\mu\nu}F^{\mu\nu}. \tag{3.15}$$

Therefore, recalling from the Eq. (3.11) that

$$\mathcal{S}_{CS}(\hat{A})=\mathcal{S}_{CS}(A)+s\left(\sum_{n=1}^{\infty}\frac{1}{m^n}\hat{\mathcal{S}}^n\right), \tag{3.16}$$

we should have

$$\frac{1}{4m}\mathrm{tr}\int d^3xF_{\mu\nu}F^{\mu\nu}\neq s\left(\sum_{n=1}^{\infty}\frac{1}{m^n}\hat{\mathcal{S}}^n\right), \tag{3.17}$$

which, of course, is in contrast with the results of the previous section which allow us in fact to express the Yang-Mills action as a pure BRST variation of a local formal power series, thereby concluding the proof of the Lemma.

The above result provides a simple cohomological understanding of the Eqs. (1.4), (1.5). It can be easily extended to cover the case in which the initial action (1.1) is supplemented with generalized terms of the Yang-Mills type. We recall in fact that among the BRST invariant actions built with $F_{\mu\nu}$ and its covariant derivatives, the Yang-Mills Lagrangian tr $F_{\mu\nu}F^{\mu\nu}$ is the term with the lowest mass dimension. Any other term of this kind will contain a higher number of $F_{\mu\nu}$ or D_μ, increasing thus its mass dimension. As a consequence, the Abelian transformations (2.3) will get modified by terms of higher order in $F^0_{\mu\nu}$ and its space-time derivatives. Therefore, provided the Chern-Simons term is included in the starting action, it will be always possible to generalize the formula (2.6), thereby expressing the field strength $F^0_{\mu\nu}$ as a BRST exact variation of a local formal power series. Everything will work as before, with the only difference that the coefficients ϑ^n_μ of the nonlinear redefinition (1.5) have now to be suitably modified. However they will remain covariant, as it will be illustrated in the following examples.

[5] We recall that the covariant character of the ϑ^n_μ's follows from the requirement of gauge invariance of $\mathcal{S}_{CS}(\hat{A})$.

IV. EXAMPLES

In order to have a better understanding of the previous results, it is useful to work out the expressions of the coefficients ϑ^n_μ in the case in which we add to the initial action (1.1) generalized terms of the Yang-Mills type. We shall study in particular the following terms:

$$\mathcal{S}_\lambda(A)=\frac{\lambda}{2m^2}\mathrm{tr}\int d^3x\varepsilon^{\mu\nu\rho}F_{\mu\sigma}D_\nu F^\sigma_\rho, \tag{4.1}$$

and

$$\mathcal{S}_\tau(A)=\frac{\tau}{4m^3}\mathrm{tr}\int d^3xF^{\mu\nu}D^2F_{\mu\nu}, \tag{4.2}$$

λ, τ being two dimensionless arbitrary parameters. The terms (4.1), (4.2) have been considered in fact by [11,12] as higher derivatives regularizing actions for the topological massive Yang-Mills.

Let us choose then as initial action the expression

$$\mathcal{S}_{YM}(A)+\mathcal{S}_{CS}(A)+\mathcal{S}_\lambda(A). \tag{4.3}$$

In this case, for the first coefficients ϑ^n_μ of the expansion (3.13) we get

$$\begin{aligned}\vartheta^1_\mu&=\tfrac{1}{4}\varepsilon_{\mu\sigma\tau}F^{\sigma\tau},\\ \vartheta^2_\mu&=\frac{(1-4\lambda)}{8}D^\sigma F_{\sigma\mu},\\ \vartheta^3_\mu&=-\frac{(1-4\lambda)}{16}\varepsilon_{\mu\sigma\tau}D^\sigma D_\rho F^{\rho\tau}\\ &\quad+\frac{g}{48}\varepsilon_{\mu\sigma\tau}[F^{\sigma\rho},F^\tau_\rho].\end{aligned} \tag{4.4}$$

Analogously, in the case in which the starting point is

$$\mathcal{S}_{YM}(A)+\mathcal{S}_{CS}(A)+\mathcal{S}_\tau(A), \tag{4.5}$$

we obtain

$$\vartheta^1_\mu=\frac{1}{4}\varepsilon_{\mu\sigma\tau}F^{\sigma\tau},$$

$$\vartheta^2_\mu=\frac{1}{8}D^\sigma F_{\sigma\mu},$$

$$\vartheta^3_\mu=-\frac{1}{16}\varepsilon_{\mu\sigma\tau}D^\sigma D_\rho F^{\rho\tau}$$

$$+\frac{g}{48}\varepsilon_{\mu\sigma\tau}[F^{\sigma\rho},F^{\tau}_{\rho}]$$

$$+\frac{\tau}{4}\varepsilon_{\mu\nu\rho}D^2F^{\nu\rho}. \qquad (4.6)$$

Of course, provided the Chern-Simons term is included in the starting action, any other combination of $\int \mathrm{tr}\, F^2$, $\int \mathrm{tr}\, FD^2F$, $\int \mathrm{tr}\, \varepsilon^{\mu\nu\rho}F_{\mu\sigma}D_\nu F^{\sigma}_{\rho}$, will lead to similar results. Notice, finally, that the expression for the coefficient ϑ^1_μ is independent from the parameters λ,τ, owing to the fact that the two terms $\mathcal{S}_\lambda(A)$ and $\mathcal{S}_\tau(A)$ in Eqs. (4.1), (4.2) are, respectively, of the order $1/m^2$ and $1/m^3$.

V. CONCLUSION

The results established in the previous sections lead us to organize the concluding remarks in two separate classes. To the first class belong the pure geometrical considerations. In the second one we discuss the field theory aspects. Here we will attempt to make contact with the known perturbative results on topological massive Yang-Mills [2,13,14,12,25,15,20]. We shall also try to provide a meaningful support to a question which arises almost naturally and which could be of great interest in order to improve our present understanding of the effective $1PI$ quantum actions of three dimensional gauge theories.

A. The geometrical set up

A very simple and attractive geometrical set up emerges from the considerations of this work. The Chern-Simons term can be interpreted in fact as a gauge invariant functional defined on the space of all possible gauge connections of the kind (3.13). Any given local Yang-Mills type action is then reproduced by evaluating the Chern-Simons functional at a specific point of this space, which amounts to a suitable choice of the gauge connection or, equivalently, of the covariant coefficients ϑ^n_μ. In this sense the Chern-Simons term may be considered as a topological generator for three-dimensional Yang-Mills gauge theories.

Moreover, this geometrical interpretation gives us a direct perception of how *rigid* can be a topological object. Of course, rigidity has here the meaning of the classical equivalence, up to local nonlinear field redefinitions, among the Yang-Mills type actions in the presence of Chern-Simons and the pure Chern-Simons term. It is worth noticing that the aforementioned rigidity of Chern-Simons has been already underlined by [9] in the framework of the consistent deformations of the master equation.

B. Field theory aspects

1. Perturbative topological massive Yang-Mills and ultraviolet finiteness

In spite of being only power counting super-renormalizable, topological massive Yang-Mills (1.1) is ultraviolet finite to all orders of perturbation theory. This property has been first observed at one loop level [2,13] and later on has been extended to all orders by combining two loops computations with finiteness by power counting at higher loops [14]. The ultraviolet finiteness has been proven in the Landau gauge; this gauge being always assumed in what follows.

In order to make use of the results established in Secs. III and IV, let us first analyze the meaning of the formulas (3.14), (3.16) from a field theory point of view. In particular, the Eq. (3.16) implies that the Yang-Mills action can be written as a pure BRST variation of a power series which contains terms of arbitrary dimension, according to the nonlinearity of the field redefinition (1.5). This could seem to be in disagreement with the standard power counting, since the BRST differential is required to act on the space of local terms of arbitrary dimension. Nevertheless, we can give a meaning to Eq. (3.16) by adopting a more general point of view and take as the starting action the formal power series

$$\mathcal{S}(A)=\mathcal{S}_{CS}(A)+\frac{1}{4m}\mathrm{tr}\int d^3x F_{\mu\nu}F^{\mu\nu}+\sum_{j=2}^{\infty}\frac{1}{m^j}\left(\sum_{k=1}^{d_k}\alpha^j_k\mathcal{S}^k_j(A)\right), \qquad (5.1)$$

where α^j_k are arbitrary coefficients and $\mathcal{S}^k_j$ are all possible higher dimensional Yang-Mills type actions built with the field strength F and the covariant derivative D. The index k in the double sum (5.1) is needed in order to account for the degeneracy (d_k) of different Yang-Mills actions with the same dimension. Notice also that, according to the results of Secs. III and IV, the action $\mathcal{S}(A)$ in Eq. (5.1) can be cast in the form of a pure Chern-Simons term, i.e. $\mathcal{S}(A)=\mathcal{S}_{CS}(\hat{A})$, with a suitable choice of the gauge connection $\hat{A}$.

For the fully quantized action in the Landau gauge, we have therefore

$$\Sigma=\mathcal{S}+\mathrm{tr}\int d^3x(b\,\partial A+\partial^\mu\bar{c}D_\mu c+A^*_\mu D^\mu c-gc^*c^2), \qquad (5.2)$$

where $b,\bar{c}$ are the Lagrangian multiplier and the antighost. The reason for this choice is that, being now the starting action Σ a formal power series, the BRST differential turns out to be naturally defined on the space of the local formal power series.

It is worth underlining that this point of view closely follows the recent new perspectives on the renormalization of gauge theories outlined in [16]. Owing to the general results on the cohomology of gauge theories [3,5], the action (5.2) is indeed renormalizable[6] in the sense that all divergences can be cancelled by the infinite terms of Σ, which spans all possible local BRST invariant Yang-Mills terms.

Although the action Σ justifies the use of the space of the formal power series for the BRST differential, we have always to face the problem of the infinite number of parameters present in the expression (5.1). However, it is easily seen that all the coefficients (m,α_k^j) correspond to BRST trivial parameters [7]. Indeed, recalling that from the results of the previous sections, every Yang-Mills type term can be written as the BRST variation of a formal power series, we immediately infer that

$$\frac{\partial\Sigma}{\partial m}=\text{BRST}-\text{variation},\qquad \frac{\partial\Sigma}{\partial\alpha_k^j}=\text{BRST}-\text{variation}, \tag{5.3}$$

the left hand side of Eq. (5.3) being understood as a formal power series. The above equations mean thus that the dependence of the action $\mathcal{S}$ from the parameters m,α_k^j can be controlled through a nonlinear redefinition of the gauge field, as showed in Secs. III and IV.

We see therefore that, in spite of the presence of an infinite number of parameters, the Eq. (5.3) implies that Σ possesses a unique nontrivial parameter[7] g, corresponding indeed to the Chern-Simons action,

$$g\,\frac{\partial\Sigma}{\partial g}=\mathcal{S}_{CS}(A)+(\text{BRST}-\text{variation}). \tag{5.4}$$

In addition, due to the topological character of Chern-Simons, it is apparent that an equation similar to Eq. (5.3) holds for the classical BRST invariant symmetric energy momentum tensor[8] $T_{\mu\nu}$ computed from $\mathcal{S}$,

$$T_{\mu\nu}=\text{BRST}-\text{variation}. \tag{5.5}$$

Furthermore, making use of the extended BRST technique [19] (see the Appendix for the details) and bearing in mind that in three dimensions there are no gauge anomalies, it follows that the Eqs. (5.3), (5.5) are easily extended at the quantum level, namely

$$\frac{\partial\Gamma}{\partial m}=\text{BRST}-\text{variation},\qquad \frac{\partial\Gamma}{\partial\alpha_k^j}=\text{BRST}-\text{variation},\qquad [T_\mu^\mu\cdot\Gamma]=\text{BRST}-\text{variation}+\text{total derivative}, \tag{5.6}$$

where, as usual, the terms in the left hand side have to be understood as BRST exact quantum insertions of formal power series (see the Appendix) and T_μ^μ stands for the trace of $T_{\mu\nu}$.

This implies, in particular, that the dependence of the quantum theory from the renormalization point μ can be controlled by the introduction of suitable BRST exact terms. These terms, being formal power series, correspond to possible nonlinear redefinitions of the fields.

The Eqs. (5.6), implicitly contained in the analysis of Ref. [1], represent our algebraic understanding of the ultraviolet finiteness properties and of the role of the nonlinear field redefinitions at the quantum level.

Recently, an independent and different proof of the ultraviolet finiteness of topological massive Yang-Mills including the vanishing of the field anomalous dimensions has been achieved by [20].

2. An open question: the fully quantum equivalence

Perhaps the most intriguing question which arises naturally in the present context is whether the classical equivalence between the massive topological Yang-Mills and the pure Chern-Simons, as stated by Eq. (1.4), can be extended at the level of the full one particle irreducible (1PI) effective action $\Gamma(A)$.

Of course, we are unable to give a satisfactory and definitive answer to this question. Only a few terms of $\Gamma(A)$ have been computed till now; $\Gamma(A)$ being an infinite series in the loop parameter expansion $\hbar$.

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[6] According to the analysis of [16], the model (5.2) can be seen as a renormalizable theory fulfilling the so called structural constraint of the type A.

[7] This situation looks rather similar to that of other kinds of well known models, as for instance the two-dimensional nonlinear sigma model [17] and the superspace four-dimensional $N=1$ super Yang-Mills [18] which, in spite of the presence of an infinite number of parameters, turn out to be characterized only by a finite set of BRST nontrivial couplings. It should be noticed also that the requirement that only a finite number of parameters are BRST nontrivial is a condition stronger than those assumed in [16]. This requirement, combined with the fulfillment of the structural constraints of [16], could give a more precise meaning to theories which are apparently powercounting nonrenormalizable.

[8] As it is well known, a symmetric classical BRST invariant energy-momentum tensor $T_{\mu\nu}$ can be obtained by the standard procedure of coupling the action $\mathcal{S}$ to gravity and set the metric to the flat one after taking the derivative of $\mathcal{S}$, i.e.

$$T_{\mu\nu}=\frac{1}{\sqrt{\eta}}\,\frac{\delta\mathcal{S}}{\delta\eta^{\mu\nu}}\bigg|_{\eta\to flat}.$$

The Chern-Simons term does not contribute to $T_{\mu\nu}$ due to the fact that it does not couple to the metric.

What we shall do here is to show that, in spite of their *nonlocal* character, a rather large number of terms contributing to $\Gamma(A)$ can be in fact reabsorbed in the Chern-Simons through *nonlocal*, but *covariant* nonlinear field redefinitions.

In what follows we shall strictly refer to the flat Euclidean space-time $\mathcal{R}^3$ endowed with the constant flat metric $\eta_{\mu\nu} = \mathrm{diag}(+,+,+)$.

We begin with a more precise definition of the 1PI quantum action. The functional $\Gamma(A)$ is meant to be the 1PI effective action computed from topological massive Yang-Mills upon quantization in the Landau gauge and after setting to zero the classical fields corresponding to the Lagrangian multiplier, ghosts and antifields. Therefore $\Gamma(A)$ depends only on the classical field A_μ defined through the Legendre transformation of the generator $\mathcal{Z}^c(\mathcal{J})$ of the connected Green's functions, i.e.

$$\Gamma(A) = \sum_{n=2}^{\infty} \int d^3x_1 \ldots . d^3x_n A(x_1) \ldots . \times A(x_n) \Gamma^n(x_{1,\ldots,}x_n), \tag{5.7}$$

where $\Gamma^n(x_{1,\ldots,}x_n)$ is the n-point[9] 1PI Green function.

Let us recall here the main basic facts about pure Chern-Simons and topological massive Yang-Mills at the quantum level which will be useful in the following:

$\Gamma(A)$ is gauge invariant and contains both local and *nonlocal* contributions at each order of perturbation theory.

For a pure Chern-Simons theory the general covariance is unbroken at the quantum level [21–24,28], implying that the dependence on the (flat) space-time metric is nonphysical. Let us also remind that, although being only powercounting renormalizable, Chern-Simons is ultraviolet finite [21–24].

As proven by [14], pure Chern-Simons is recovered as the infinite mass limit $m \to \infty$ of topological massive Yang-Mills.

In spite of the presence of the antisymmetric tensor $\varepsilon_{\mu\nu\rho}$, explicit regularizations preserving the BRST invariance have been constructed for topological massive Yang-Mills [14,12,25]. Although not needed for an algebraic analysis, the existence of regularizations preserving the BRST symmetry is rather important in order to carry out computations. This means that the BRST invariance can be maintained manifestly at each step, implying that $\Gamma(A)$ can be constructed without leaving the space of the gauge invariant terms.

Let us now focus on the structure of the 1PI effective action $\Gamma(A)$. Needless to say, $\Gamma(A)$ is expected to be a quite complicated object. Moreover, because of the gauge invariance, $\Gamma(A)$ will certainly contain a large number of terms built with the field strength F and its covariant derivatives, intertwined in very complicated nonlocal, but gauge invariant combinations [13].

[9]We have not specified the group indices in Eq. (5.7) since they are not relevant for the forthcoming considerations.

However, it is very simple to see that if we start with a gauge invariant nonlocal action built with F and its covariant derivatives in the presence of Chern-Simons, the recursive formula (2.6) can be suitably adapted to the nonlocal case, with the result that the nonlocal term can be reabsorbed in the pure Chern-Simons through a *nonlocal* field redefinition. Remarkably in half and in spite of the nonlocality, the redefined gauge field will still transform as a connection, due to the fact that the coefficients entering the nonlocal redefinition turn out to be covariant, as it will be shown in the next example. Thus, a great amount of nonlocal quantum effects coming from nonlocal terms built with F can be taken into account by interpreting the Chern-Simons as a gauge invariant functional defined on the space of the gauge connections of the type $\hat{A} = A + \delta A$, where δA contains now both local and nonlocal covariant terms, $\delta A = \delta A^{loc} + \delta A^{nloc}$.

In order to have a better feeling of how things go in the nonlocal case, let us compute the first coefficients $\vartheta^1_\mu, \vartheta^2_\mu$ for the nonlocal action

$$\mathcal{S}^{nloc}_{YM}(A) = \frac{1}{4m} \int d^3x d^3y F^2(x) |x-y| F^2(y), \tag{5.8}$$

with

$$F^2(x) = \mathrm{tr}\, F^{\mu\nu}(x) F_{\mu\nu}(x). \tag{5.9}$$

The expression (5.8) is one of the simplest nonlocal invariant terms depending on F which is expected to appear in the loop expansion of $\Gamma(A)$. The coefficients $\vartheta^1_\mu, \vartheta^2_\mu$ are easily found to be

$$\vartheta^1_\mu = \tfrac{1}{4} \varepsilon_{\mu\nu\rho} F^{\nu\rho}(x) \int d^3y |x-y| F^2(y),$$

$$\vartheta^2_\mu = -\tfrac{1}{8} \left(\int d^3y \left| x-y \right| F^2(y) \right) \times D^x_\sigma \int d^3z F^\sigma_\mu(x) |x-z| F^2(z), \tag{5.10}$$

D^x_σ being the covariant derivative acting on the point x. Thus

$$\mathcal{S}_{CS}(A) + \mathcal{S}^{nloc}_{YM}(A) = \mathcal{S}_{CS}(\hat{A}) + O(1/m^3), \tag{5.11}$$

with

$$\hat{A}_\mu = A_\mu + \frac{1}{m} \vartheta^1_\mu + \frac{1}{m^2} \vartheta^2_\mu + O(1/m^3). \tag{5.12}$$

As anticipated, the coefficients $\vartheta^1_\mu, \vartheta^2_\mu$ in Eq. (5.10), although nonlocal, transform covariantly,[10] so that the rede-

[10]Perhaps the covariant character of the coefficients ϑ_μ in the nonlocal case could be understood by noticing that the Lemma of the Sec. III, being of a purely geometrical nature, could be in principle applied to nonlocal actions built up with F and its covariant derivatives.

fined field $\hat{A}_\mu$ is *still a connection*. It is worth emphasizing that the form of the space-time function between $F^2(x)$ and $F^2(y)$ in the Eq. (5.8) is in fact completely irrelevant. More sophisticated examples of nonlocal F-dependent actions can be worked out, leading to similar results. We see therefore that a large class of nonlocal terms can be reabsorbed in the pure Chern-Simons. In our opinion this observation is a signal of the fact that the aforementioned rigidity of the topological Chern-Simons may persist at the quantum level. Although we have a solid understanding of the BRST cohomology in the space of the local functionals [3–5], the situation is completely different in the nonlocal case. Up to our knowledge there is no proof of the fact that the nonlocal terms are built essentially with the field strength and its derivatives. Notice that we are not demanding here the complete characterization of the nonlocal terms, including the knowledge of the space-time dependence of the 1PI n-point Green function $\Gamma^n(x_{1,\ldots,}x_n)$. A weaker characterization guaranteeing the simple presence of F would be almost sufficient in order to establish the quantum equivalence between $\Gamma(A)$ and $\mathcal{S}_{CS}(A)$. It would be a very nice event if in the case of the flat space-time $\mathcal{R}^3$, the 1PI effective action could be reset to a pure Chern-Simons, up to nonlinear field redefinitions.

Let us conclude this section by drawing a possible path in favor of this hypothesis. The forthcoming considerations heavily rely on a rather appealing suggestion of [13] (see in particular Sec. IV) and on the explicit one and two loop computations on topological massive Yang-Mills and on pure Chern-Simons done till now [2,13,14,12,25,21,22].

After having reabsorbed all the nonlocal terms that we can, we should be able to write the complete 1PI effective action $\Gamma(A)$ in the following form:

$$\Gamma(A)=\zeta\mathcal{S}_{CS}(\hat{A})+\Xi, \tag{5.13}$$

with

$$\hat{A}=A+\delta A^{loc}+\delta A^{nloc}. \tag{5.14}$$

The coefficient[11] ζ in Eq. (5.13) is a power series in $\hbar$ accounting for possible finite corrections to the Chern-Simons term itself and can be reabsorbed by a further *finite multiplicative* redefinition of the gauge field and of the coupling constant g.

The extra term Ξ in Eq. (5.13) represents all the gauge invariant nonlocal terms which cannot be reabsorbed through nonlinear field redefinitions. However Ξ should be very constrained. It should not contain m since, due to the Eqs. (5.6),[12] m-dependent terms are expected to be related to nonlinear redefinitions. Then Ξ should survive then the infinite mass limit $m\rightarrow\infty$. Pure Chern-Simons considerations should thus apply. Therefore, according to [13] (see Sec. IV), Ξ should be vanishing, due to a parity argument [13] and to the general covariance of the Chern-Simons in the Landau gauge [21–24,28]. In summary, the 1PI effective action of topological Yang-Mills in flat space-time $\mathcal{R}^3$ should be resummed to pure Chern-Simons, up to nonlinear field redefinitions. This behavior may be completely different for a generic curved three manifold, as other kinds of topological invariants like the Ray-Singer torsion [26] are expected to appear.

It is rather important to underline here that a nonlocal field redefinition has been in fact already used by [25] in order to reset the one loop effective action of Chern-Simons in the light cone gauge to a pure Chern-Simons action.

Other kinds of three-dimensional effective actions, as for instance the fermionic determinant of a two component massive spinor could be contemplated. Indeed, the infinite mass limit of the Abelian fermionic determinant in flat space-time is nothing, but pure Chern-Simons [27]. Moreover, the perturbative expression obtained so far for the Abelian determinant can be easily reabsorbed through nonlinear redefinitions [29] into a pure Chern-Simons, providing thus a further evidence

C. Final remarks

We emphasize once again that the interpretation of the Chern-Simons as a gauge invariant functional which is able to reproduce any given *local* Yang-Mills type action is rather attractive.

Whether this pure geometrical set up can be extended at the level of the full 1PI effective quantum action is still an open question. In any case a great part of $\Gamma(A)$ can be certainly reset to a pure Chern-Simons term up to nonlinear field redefinitions.

It is worth underlining that the covariant character of the coefficients ϑ_μ (in both local and nonlocal case) which allows to interpret the redefined field $\hat{A}_\mu$ as a gauge connection could shed some light on the role of the nonlinear field redefinitions in quantum field theory. This point could be of a certain relevance within the new recent perspectives on the renormalization of gauge theories [16].

The present results naturally remind us the relationship with general relativity. After all, being $\hat{A}_\mu$ a connection, the resulting action $\mathcal{S}_{CS}(\hat{A})$ is gauge invariant and looks as good as the initial one $\mathcal{S}_{CS}(A)$. This suggests that, as far as the gauge invariance is taken as the guide principle in order to select appropriate actions, we should still have the freedom of choosing the connection.

The inclusion of matter fields is under investigation.

We hope, finally, that this work will be of some help in order to improve our present understanding of three-dimensional gauge theories and of the role of the topological actions.

[11]There has been a long discussion in the last years about a possible universal meaning of ζ. This question being not addressed here, we remind the original literature [11,12,22].

[12]It is useful to recall here that the action Σ in Eq. (5.2) differs from the pure massive topological Yang-Mills by the infinite set of gauge parameters α_k^j.

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APPENDIX

1. Extended BRST technique

The extended BRST technique [19] is a very powerful tool which allows us to control the dependence of the theory at the quantum level from parameters associated to exact BRST terms. Let us present here how this technique works in the case of the parameter m of the action Σ in Eq. (5.2):

$$\Sigma=\mathcal{S}+\mathrm{tr}\int d^3x(b\partial A+\partial^{\mu}\bar{c}D_{\mu}c+A^{*}_{\mu}D^{\mu}c-gc^{*}c^{2}), \tag{A1}$$

with

$$\mathcal{S}(A)=\mathcal{S}_{CS}(A)+\frac{1}{4m}\mathrm{tr}\int d^3xF_{\mu\nu}F^{\mu\nu}+\sum_{j=2}^{\infty}\frac{1}{m^j}\left(\sum_{k=1}^{d_k}\alpha^j_k\mathcal{S}^k_j(A)\right). \tag{A2}$$

The action Σ obeys the classical Slavnov-Taylor identity

$$\mathrm{tr}\int d^3x\left(\frac{\delta\Sigma}{\delta A_{\mu}}\frac{\delta\Sigma}{\delta A^{*\mu}}+\frac{\delta\Sigma}{\delta c}\frac{\delta\Sigma}{\delta c^{*}}+b\frac{\delta\Sigma}{\delta\bar{c}}\right)=0, \tag{A3}$$

from which it follows that the so called linearized operator $\mathcal{B}_{\Sigma}$ defined as

$$\mathcal{B}_{\Sigma}=\mathrm{tr}\int d^3x\left(\frac{\delta\Sigma}{\delta A_{\mu}}\frac{\delta}{\delta A^{*\mu}}+\frac{\delta\Sigma}{\delta A^{*\mu}}\frac{\delta}{\delta A_{\mu}}+\frac{\delta\Sigma}{\delta c}\frac{\delta}{\delta c^{*}}+\frac{\delta\Sigma}{\delta c^{*}}\frac{\delta}{\delta c}+b\frac{\delta}{\delta\bar{c}}\right), \tag{A4}$$

is nilpotent

$$\mathcal{B}_{\Sigma}\mathcal{B}_{\Sigma}=0. \tag{A5}$$

As it is well known [7], this operator identifies the full BRST differential acting on the fields and antifields.

Owing to the results of Secs. III and IV, we have

$$\frac{\partial\Sigma}{\partial m}=\mathcal{B}_{\Sigma}\Lambda, \tag{A6}$$

Λ being an integrated local formal power series in the fields and antifields of ghost number -1. According to [19], we introduce the term Λ in the classical action Σ by means of a constant parameter ξ of ghost number 1, namely

$$\tilde{\Sigma}=\Sigma+\xi\Lambda. \tag{A7}$$

Let us now compute the quantity

$$\mathrm{tr}\int d^3x\left(\frac{\delta\tilde{\Sigma}}{\delta A_{\mu}}\frac{\delta\tilde{\Sigma}}{\delta A^{*\mu}}+\frac{\delta\tilde{\Sigma}}{\delta c}\frac{\delta\tilde{\Sigma}}{\delta c^{*}}+b\frac{\delta\tilde{\Sigma}}{\delta\bar{c}}\right). \tag{A8}$$

The expression (A8) is expected to be nonvanishing, due to the use of the modified action $\tilde{\Sigma}$. However, due to the fact that $\xi\xi=0$, we easily get

$$\mathrm{tr}\int d^3x\left(\frac{\delta\tilde{\Sigma}}{\delta A_{\mu}}\frac{\delta\tilde{\Sigma}}{\delta A^{*\mu}}+\frac{\delta\tilde{\Sigma}}{\delta c}\frac{\delta\tilde{\Sigma}}{\delta c^{*}}+b\frac{\delta\tilde{\Sigma}}{\delta\bar{c}}\right)=-\xi\mathcal{B}_{\Sigma}\Lambda. \tag{A9}$$

Therefore from

$$\xi\mathcal{B}_{\Sigma}\Lambda=\xi\frac{\partial\Sigma}{\partial m}=\xi\left(\frac{\partial\tilde{\Sigma}}{\partial m}-\xi\frac{\partial\Lambda}{\partial m}\right)=\xi\frac{\partial\tilde{\Sigma}}{\partial m}, \tag{A10}$$

it follows that the action $\tilde{\Sigma}$ satisfies the modified Slavnov-Taylor identity

$$\int d^3x\ \mathrm{tr}\left(\frac{\delta\tilde{\Sigma}}{\delta A_{\mu}}\frac{\delta\tilde{\Sigma}}{\delta A^{*\mu}}+\frac{\delta\tilde{\Sigma}}{\delta c}\frac{\delta\tilde{\Sigma}}{\delta c^{*}}+b\frac{\delta\tilde{\Sigma}}{\delta\bar{c}}\right)+\xi\frac{\partial\tilde{\Sigma}}{\partial m}=0. \tag{A11}$$

This expression is easily recognized to be of the type of an extended Slavnov-Taylor identity [19]. In fact, from a cohomological point of view, the parameters ξ,m form a doublet, i.e.

$$\mathcal{B}_{\tilde{\Sigma}}m=\xi,\quad \mathcal{B}_{\tilde{\Sigma}}\xi=0. \tag{A12}$$

The absence of gauge anomalies in three dimensions (see Subsec. II B) guarantees thus that the identity (A11) holds at the quantum level

$$\int d^3x\ \mathrm{tr}\left(\frac{\delta\tilde{\Gamma}}{\delta A_{\mu}}\frac{\delta\tilde{\Gamma}}{\delta A^{*\mu}}+\frac{\delta\tilde{\Gamma}}{\delta c}\frac{\delta\tilde{\Gamma}}{\delta c^{*}}+b\frac{\delta\tilde{\Gamma}}{\delta\bar{c}}\right)+\xi\frac{\partial\tilde{\Gamma}}{\partial m}=0. \tag{A13}$$

Acting now on the expression (A13) with the test operator $\partial/\partial\xi$ and setting ξ to zero, we immediately get

$$\frac{\partial\Gamma}{\partial m}=\mathcal{B}_{\Gamma}[\Lambda\cdot\Gamma], \tag{A14}$$

$$\Gamma=\tilde{\Gamma}|_{\xi=0},$$

thereby proving the statement (5.6). The same procedure applies to the parameters α_k^j as well as to the energy-momentum tensor. In the latter case the extended BRST technique has to be done twice. First one considers the integrated insertion $\int d^3x[T_\mu^\mu\cdot\Gamma]$, for which one gets

$$\int d^3x[T_\mu^\mu\cdot\Gamma]=\text{BRST}-\text{variation}. \tag{A15}$$

A further application of the extended BRST technique with local space-time dependent parameters allows to obtain the final result (5.6).

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